
Analysis and Approximation of Optimal Co-Scheduling on Chip Multiprocessors

Yunlian Jiang

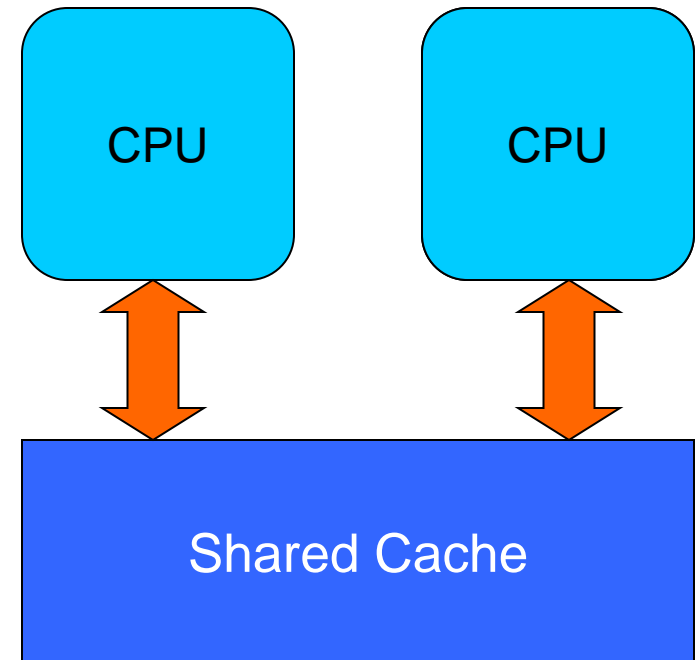
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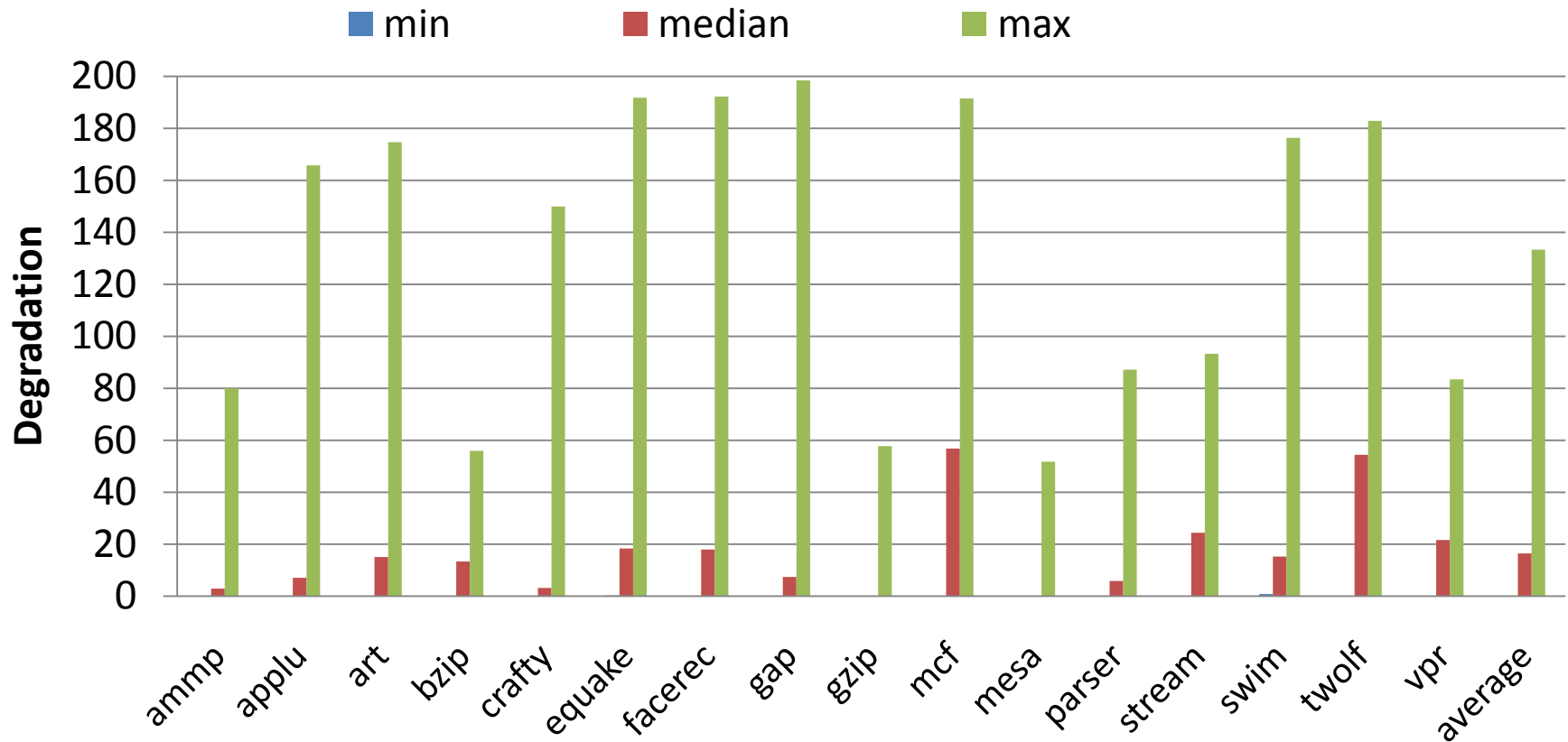
Cache Sharing on CMP

- Shorten inter-thread communication
- Flexible usage of cache
 - degrade performance
 - impair fairness
 - hurt performance isolation



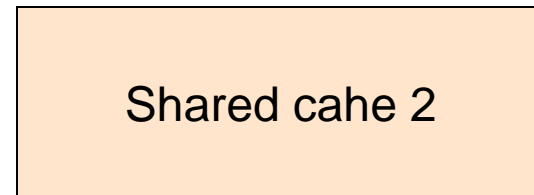
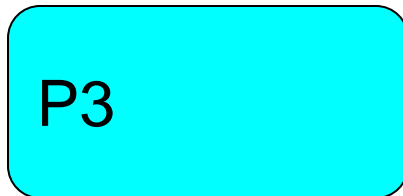
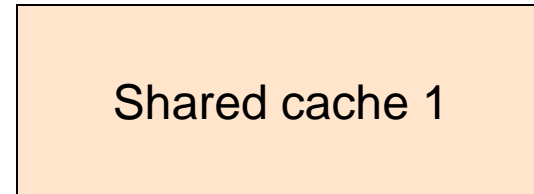
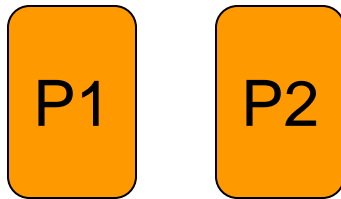
Degradation is affected by co-runner

Performance degradation range



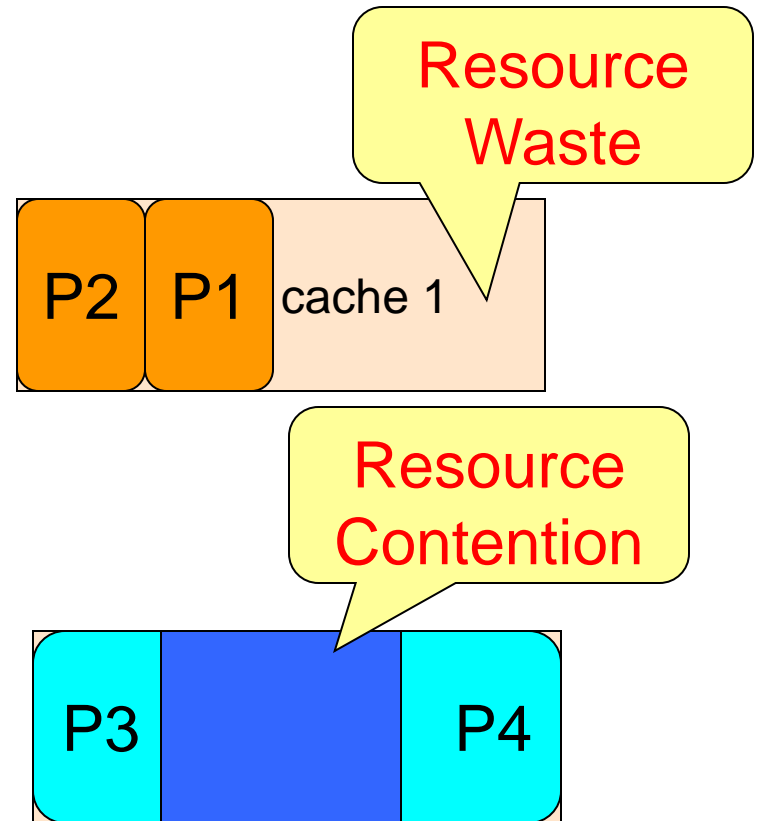
Job Co-Scheduling

- To assign jobs to chips in a manner to minimize contention



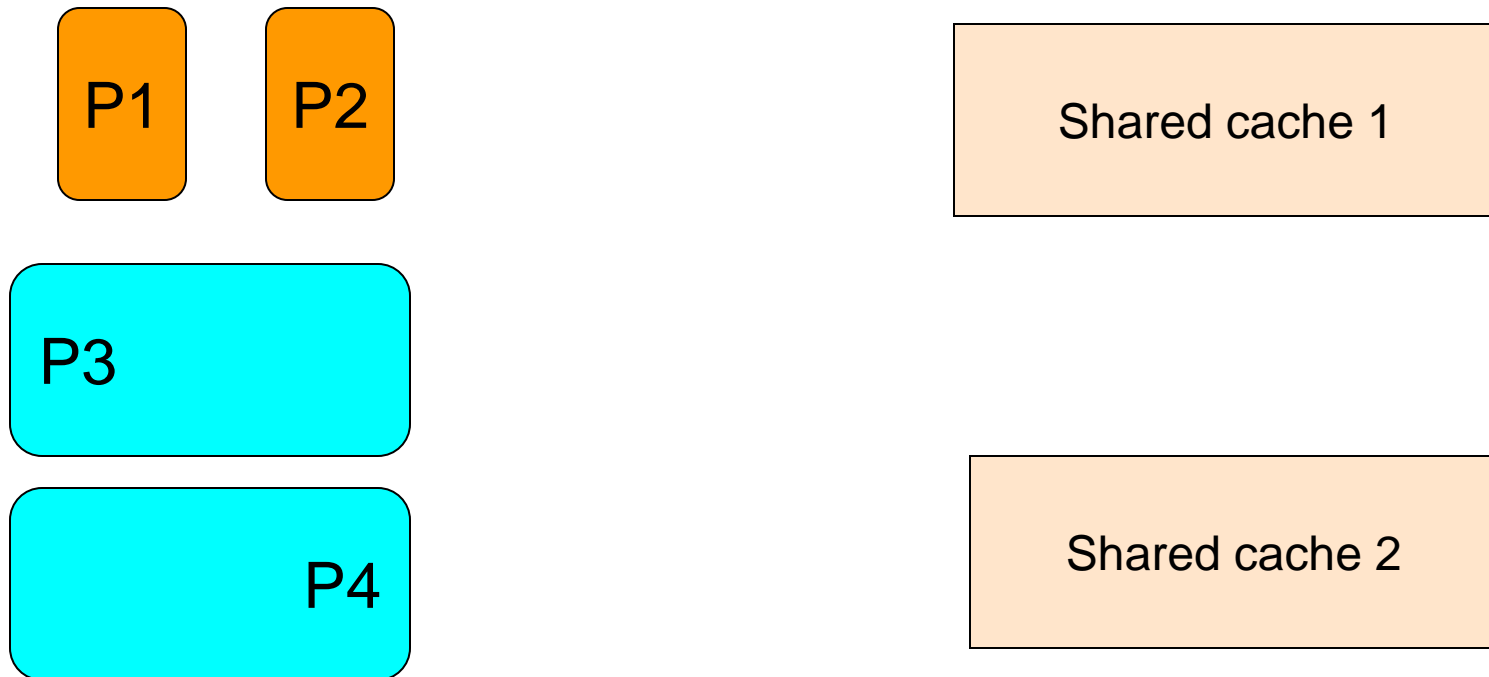
Job Co-Scheduling

- To assign jobs to chips in a manner to minimize contention



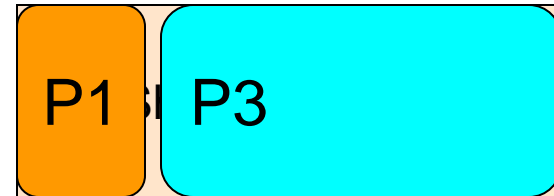
Job Co-Scheduling

- To assign jobs to chips in a manner to minimize contention



Job Co-Scheduling

- To assign jobs to chips in a manner to minimize contention



The Goal of this Work

- Related work
 - Snavelly etc. [00' ASPLOS]
- Goal of this work
 - Find the optimal schedule on CMP system
- Benefits
 - Evaluate current schedule quality
 - Applied in real system

Contributions

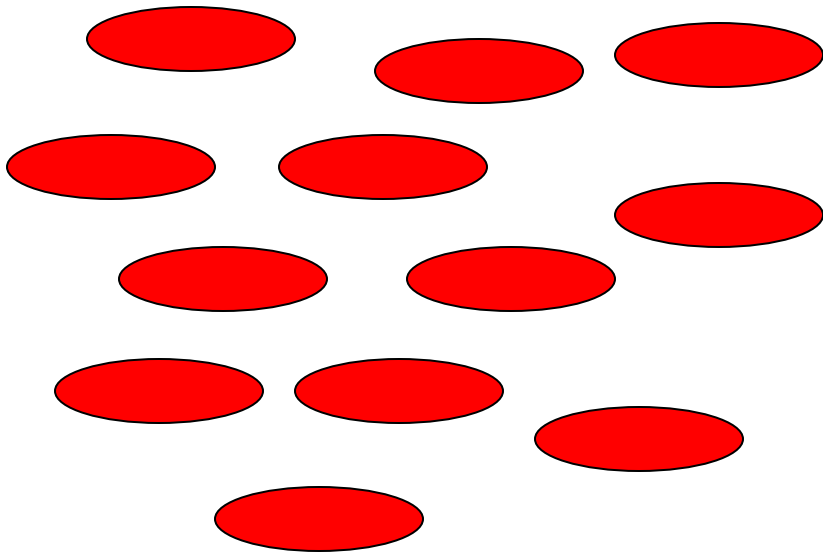
- Polynomial optimal solution on Dual-core systems
- NP-Completeness proof on K-core ($K > 2$) systems
- Polynomial approximation algorithms on K-core ($K > 2$) systems

Contributions

- Polynomial optimal solution on Dual-core systems
- NP-Completeness proof on K-core ($K > 2$) systems
- Polynomial approximation algorithms on K-core ($K > 2$) systems

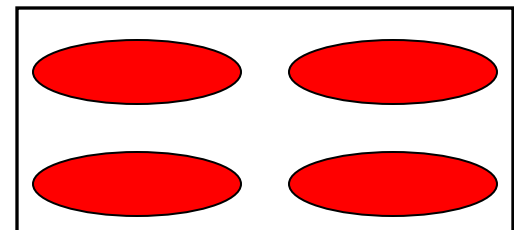
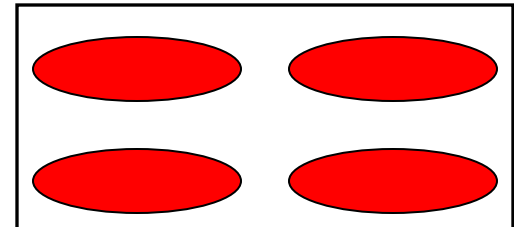
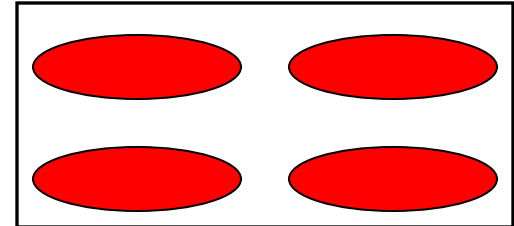
Problem Formulation

- M jobs
- N Core processors



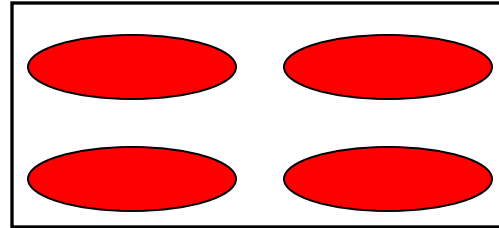
Problem Formulation

- M jobs
- N Core processors



Problem Formulation

- Assignment



$$Deg_i = \frac{cCPI_i - sCPI_i}{sCPI_i}$$

- Goal Minimize $\sum Deg_i$

Dual-Core System

- Polynomial Solution

- Minimum-weight perfect matching

[Edmonds: 1965]

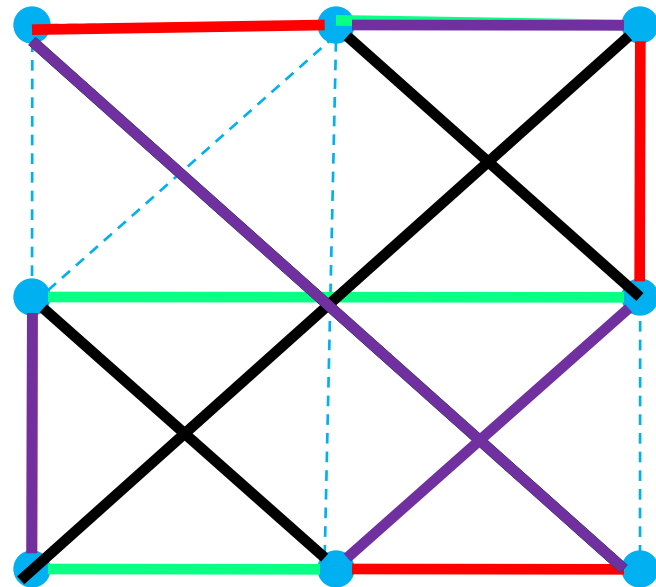
- Matching

- A matching M in graph G is a set of edges with no common vertex.

- perfect matching is a matching which matches all vertices of the graph

Dual-Core System

- Minimum-weight perfect matching
 - In edge weighted graph
 - Sum of weight of edges in the match is minimum



Dual-Core System

■ Job

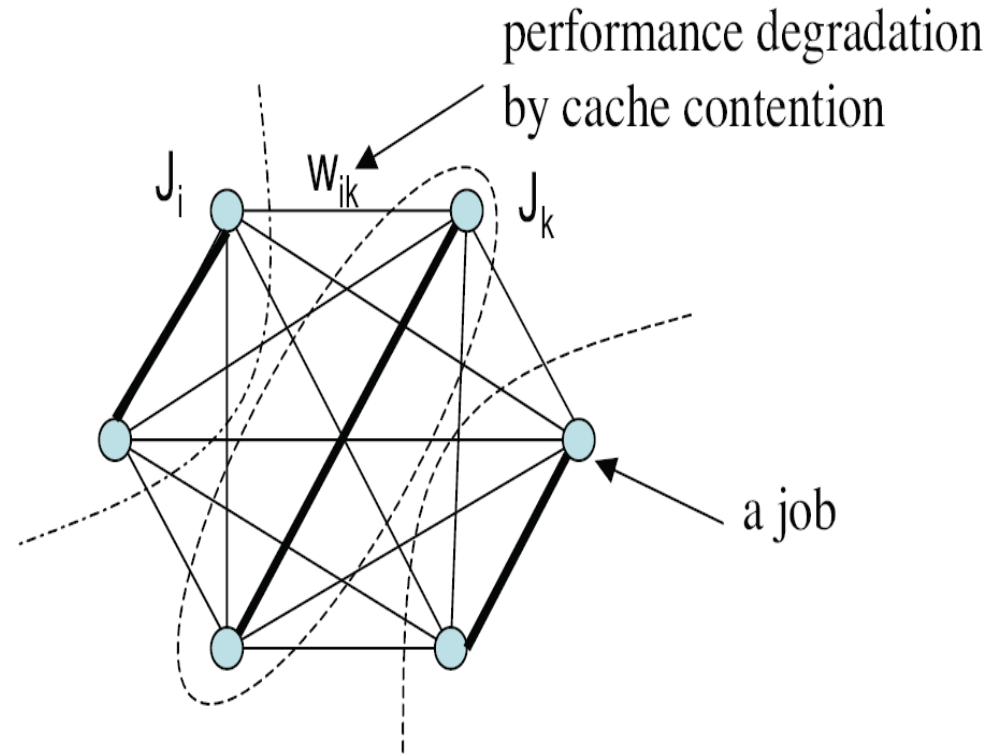
➤ Nodes

■ Corun-Degradation

➤ Edge Weight

■ Optimal Schedule

➤ Minimum weight perfect matching



Contributions

- Polynomial optimal solution on Dual-core systems
- **NP-Completeness proof on K-core ($K > 2$) systems**
- Polynomial approximation algorithms on K-core ($K > 2$) systems

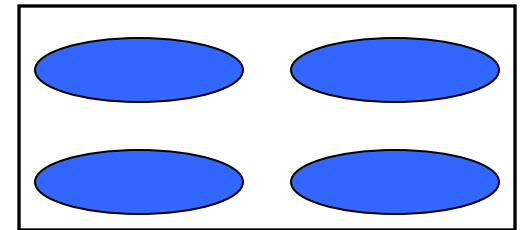
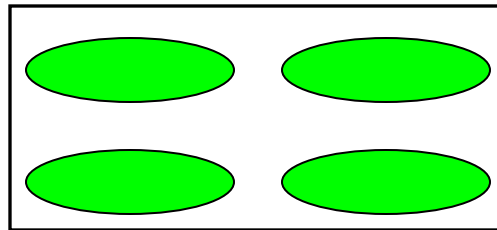
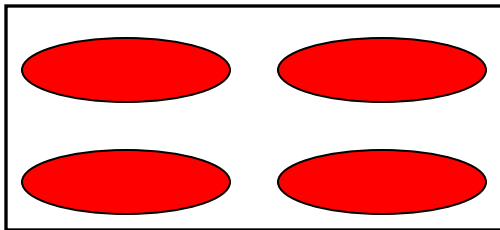
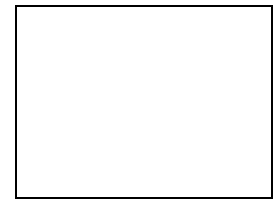
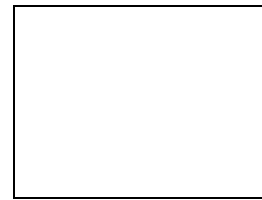
NP-Completeness proof

- NP proof
 - Given a schedule, can compute

- Reduction
 - NP-Complete problem \rightarrow Job Co-scheduling
 - Multidimensional Assignment Problem (MAP)

NP-Completeness Proof

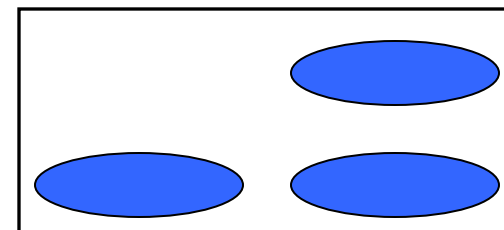
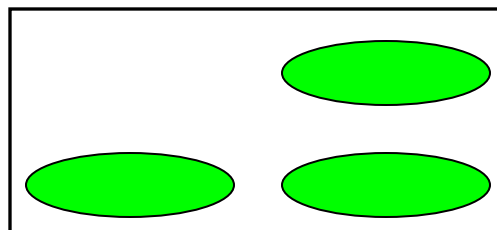
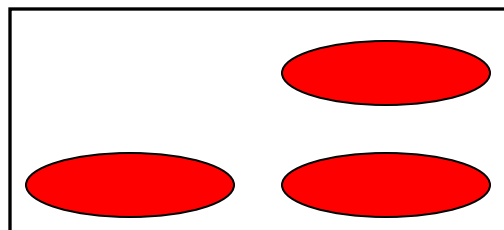
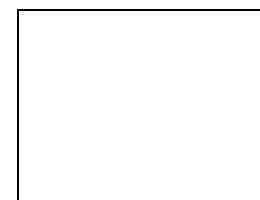
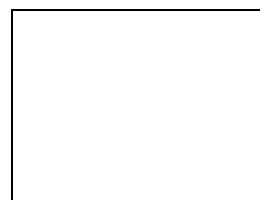
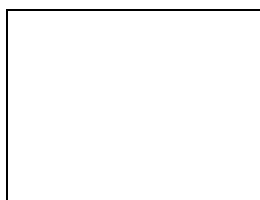
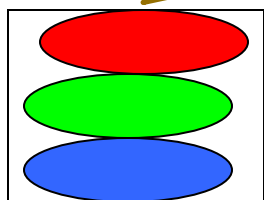
- MAP



NP-Completeness Proof

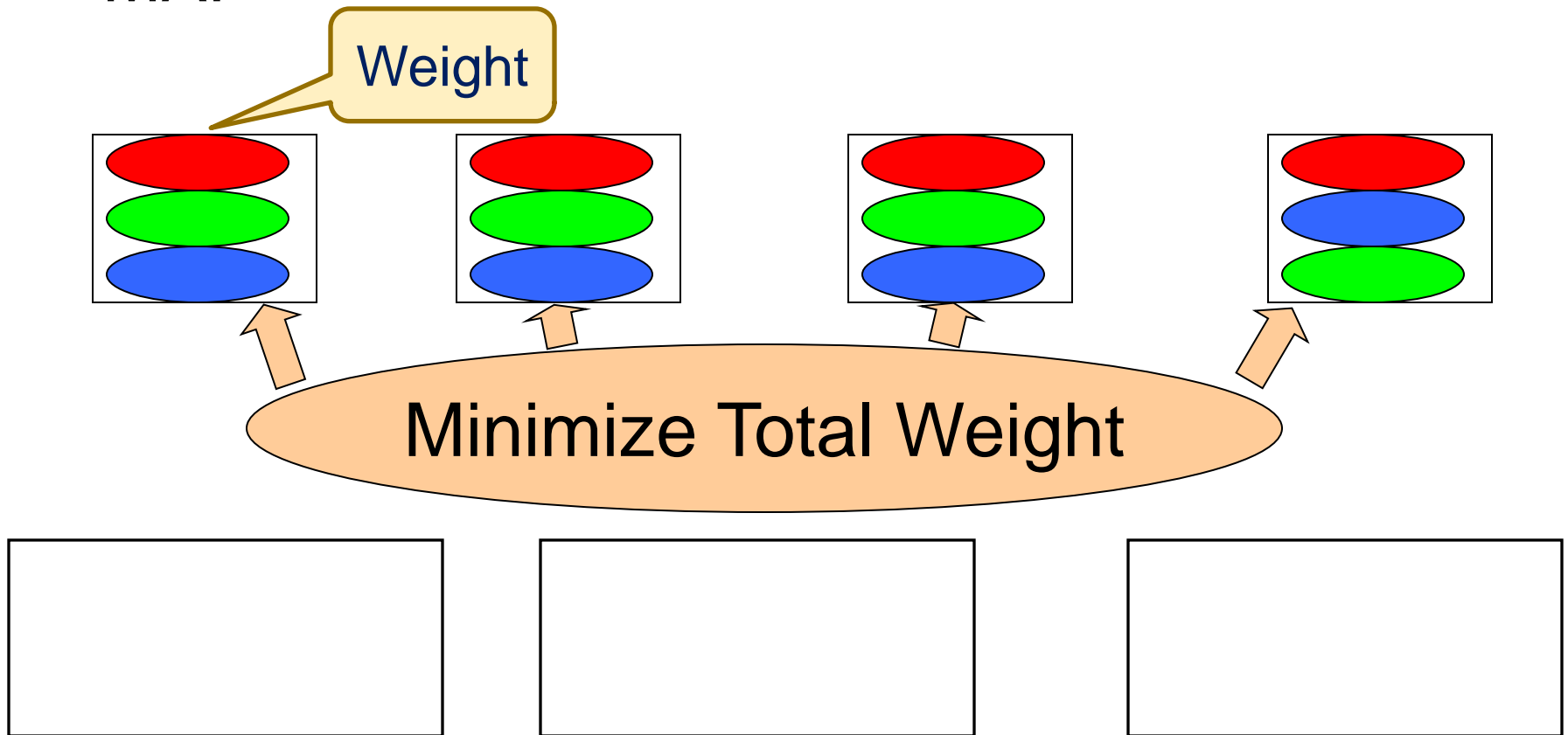
■ MAP

Weight



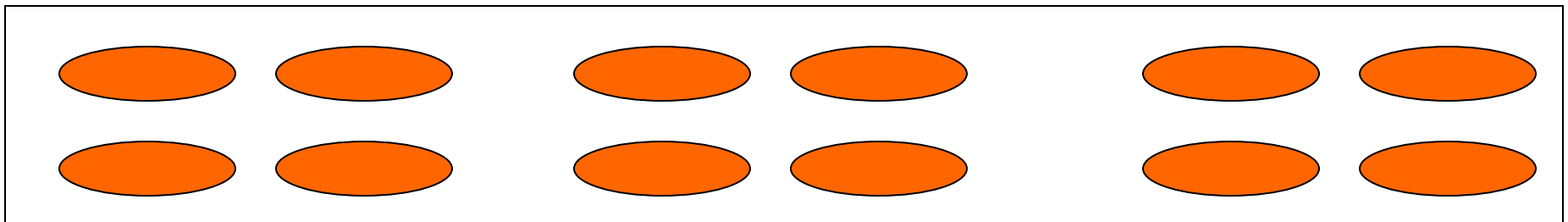
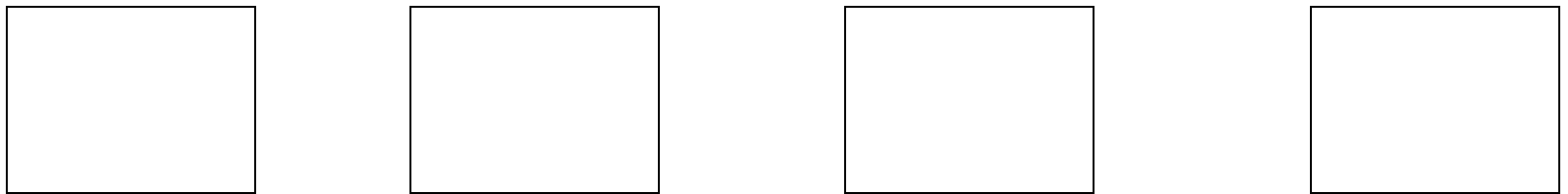
NP-Completeness Proof

■ MAP



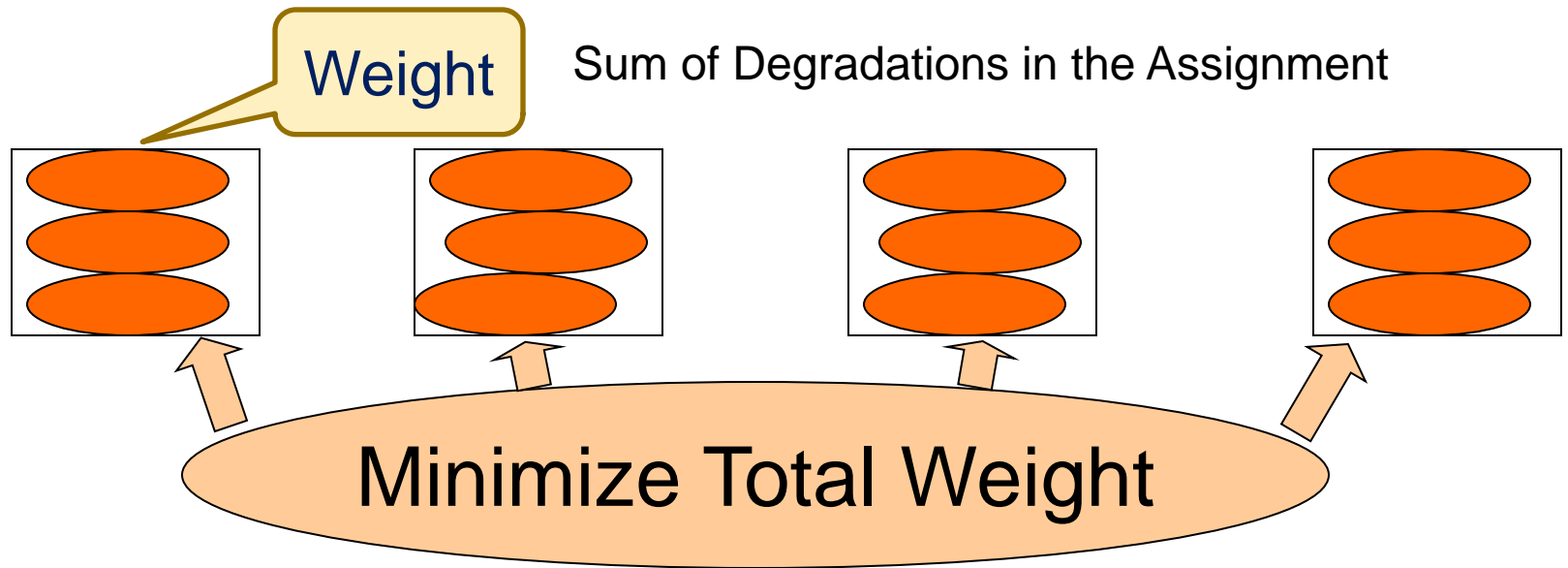
NP-Completeness Proof

- Job Co-Scheduling on CMP



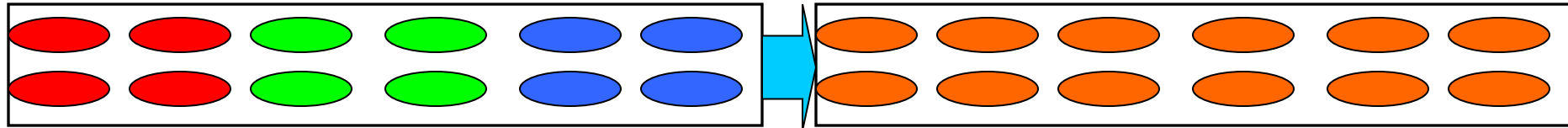
NP-Completeness Proof

■ Job Co-Scheduling on CMP



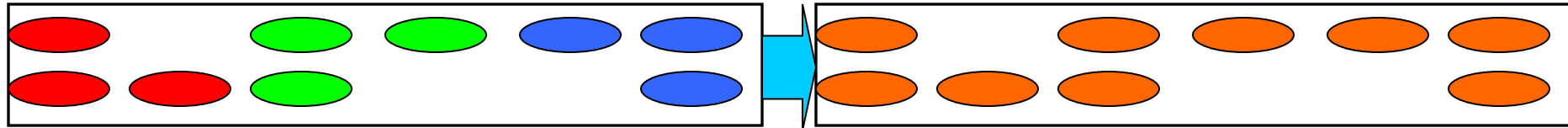
NP-Completeness Proof

- MAP \rightarrow Job Co-Scheduling

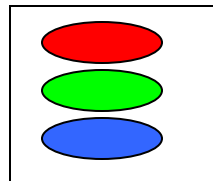


NP-Completeness Proof

■ MAP \rightarrow Job Co-Scheduling

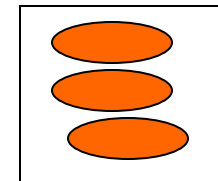


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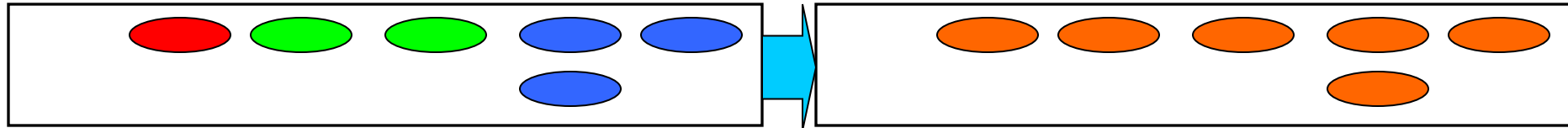
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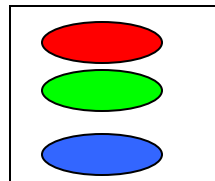


NP-Completeness Proof

■ MAP \rightarrow Job Co-Scheduling

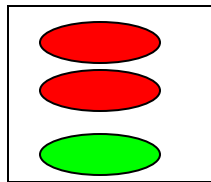
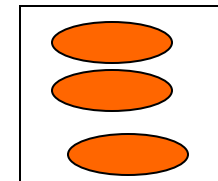


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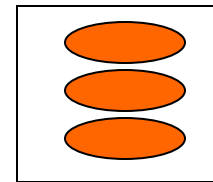


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Weight



Weight



= ∞

Contributions

- Polynomial optimal solution on Dual-core systems
- NP-Completeness proof on K-core ($K > 2$) systems
- Polynomial approximation algorithms on K-core ($K > 2$) systems

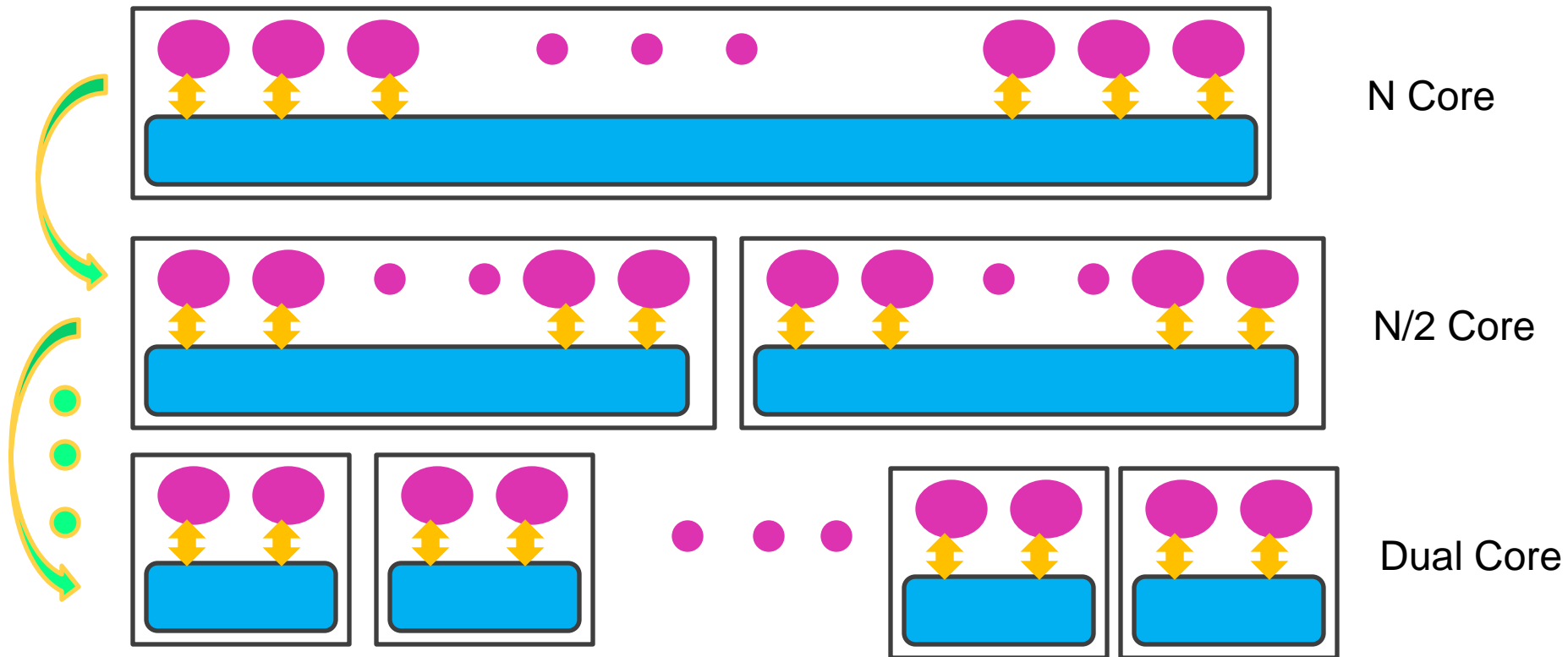
Approximation algorithms

- Hierarchical Perfect Matching

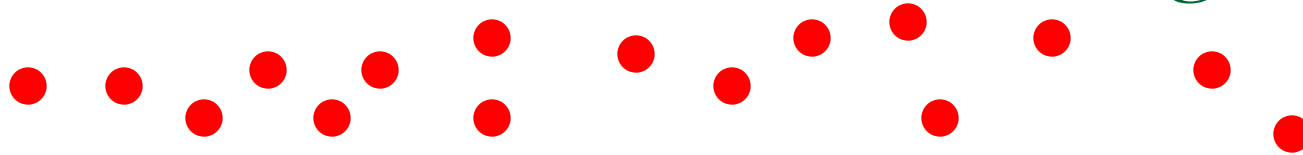
- Greedy

Hierarchical Perfect Matching

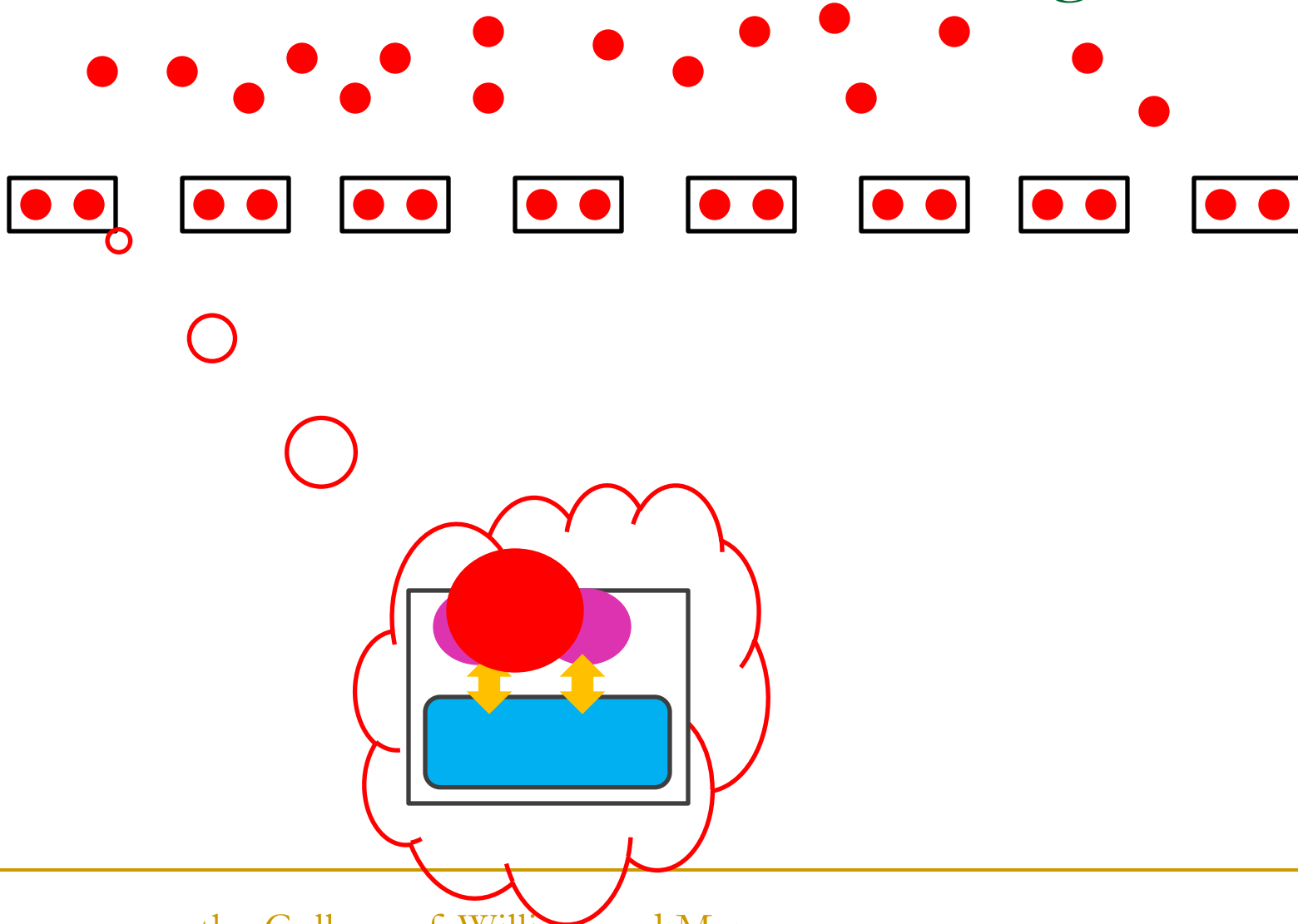
- Dual-core system optimal solution



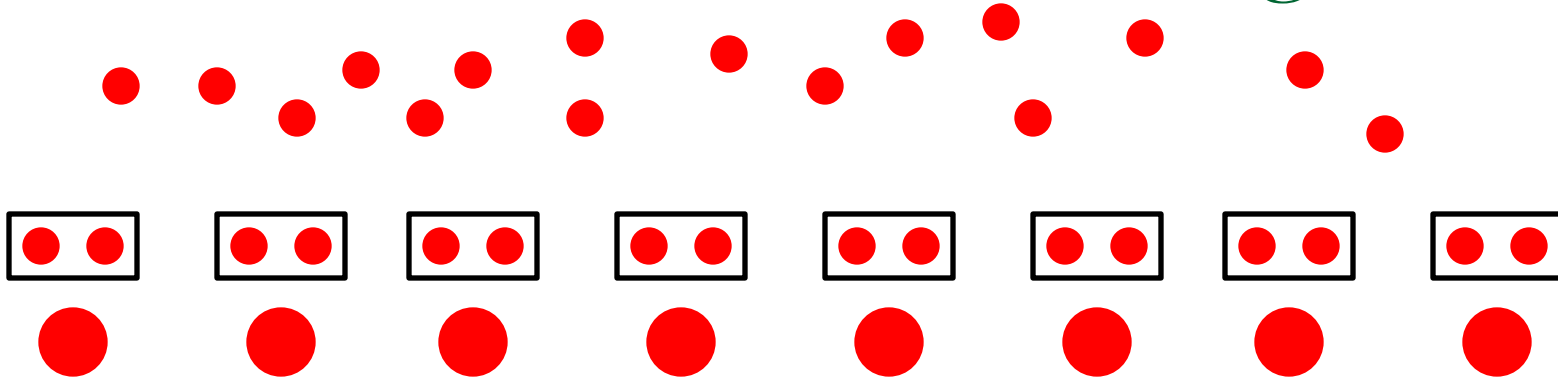
Hierarchical Perfect Matching



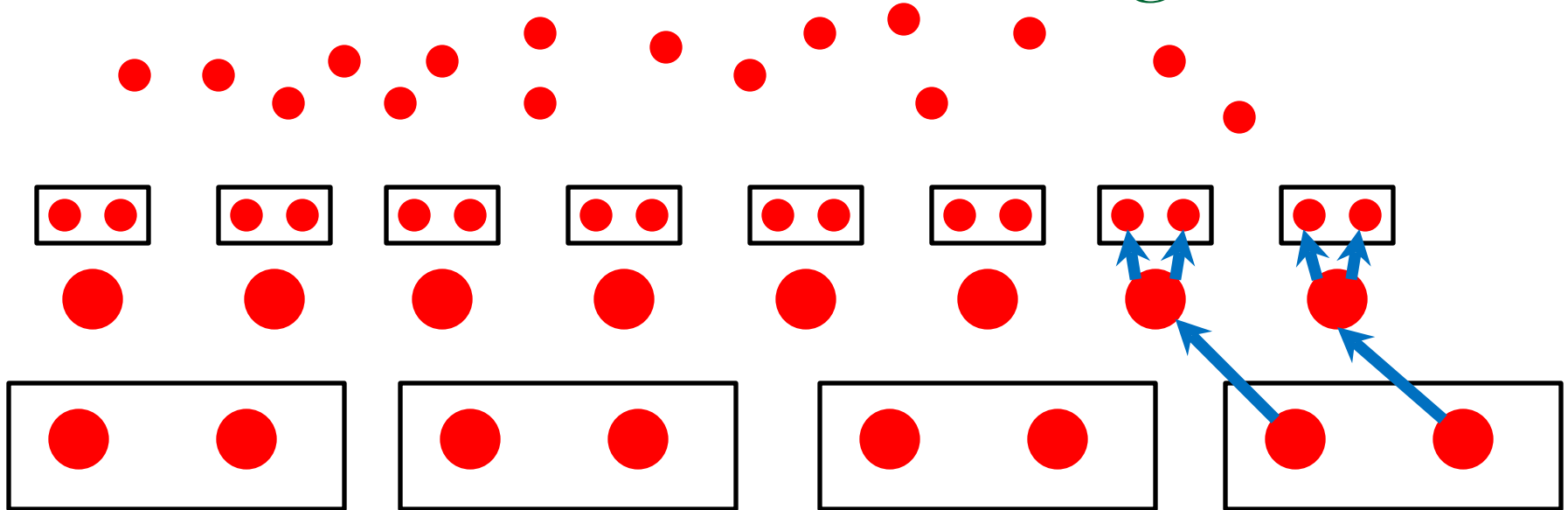
Hierarchical Perfect Matching



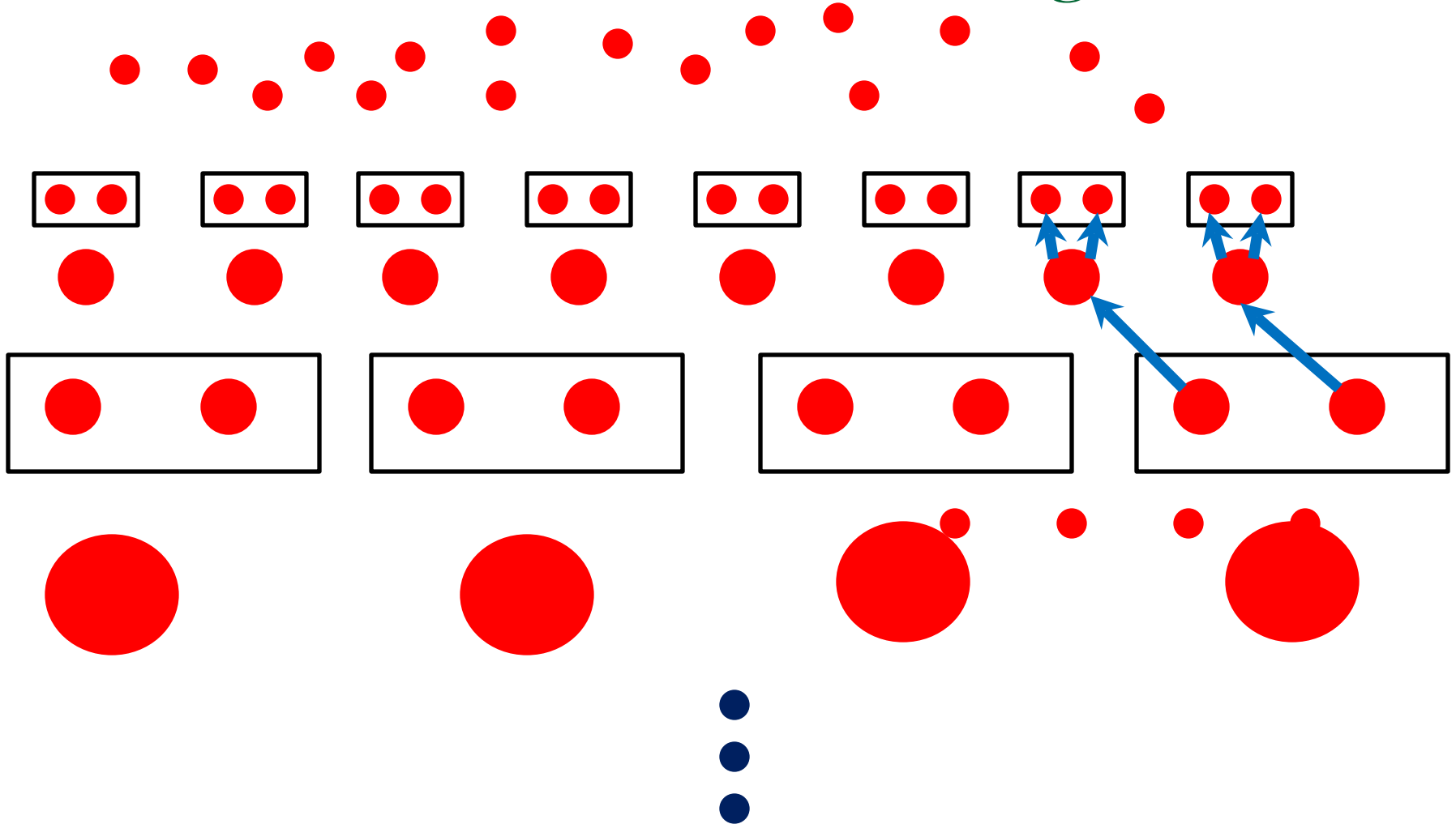
Hierarchical Perfect Matching



Hierarchical Perfect Matching



Hierarchical Perfect Matching



Greedy Algorithm

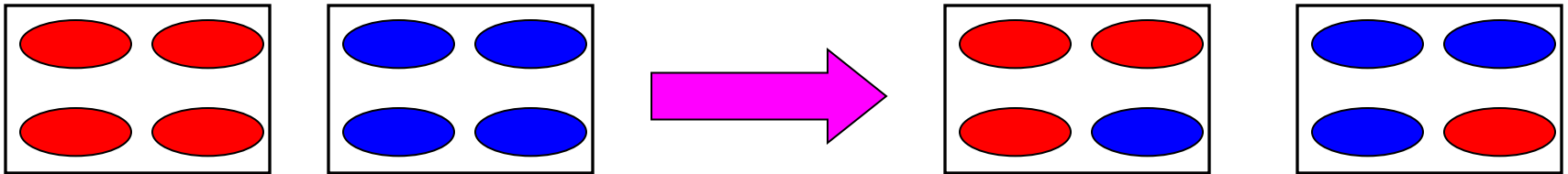
- Basic idea
 - Schedule the least “polite” job first
- “politeness” of a Job
 - Sum of degradations of all the assignments contain this job.
 - Impact of a job on others

Greedy Algorithm

- I. Sort unassigned jobs based on politeness
- II. Pick the least politeness job J to schedule
- III. Add assignment contains J with least degradation to schedule
- IV. Update unassigned job list

Local Optimization

■ Main Scheme



for $i \leftarrow 1$ to $K-1$

K: number of assignments

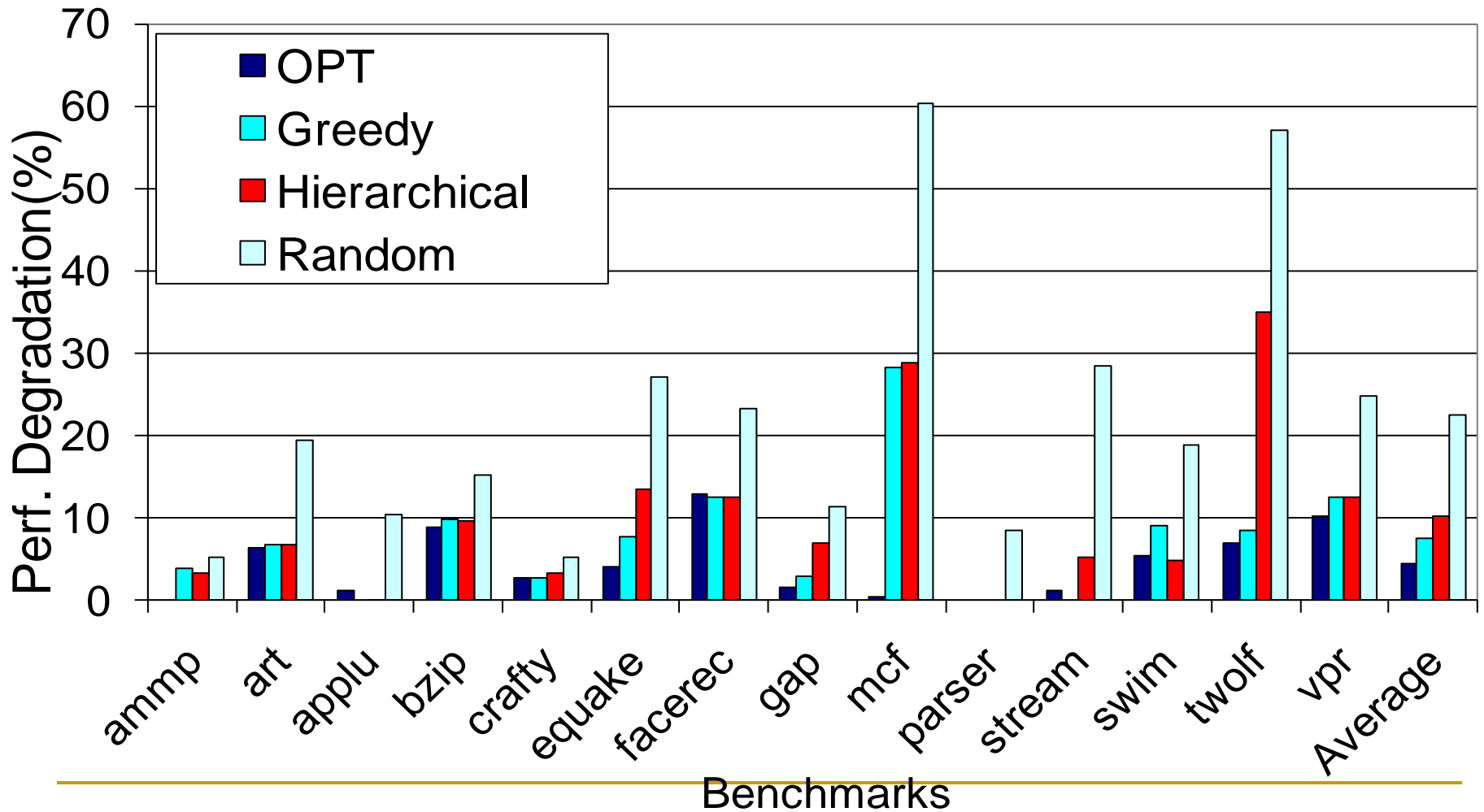
for $j \leftarrow i+1$ to K

Local-Optimization(i, j)

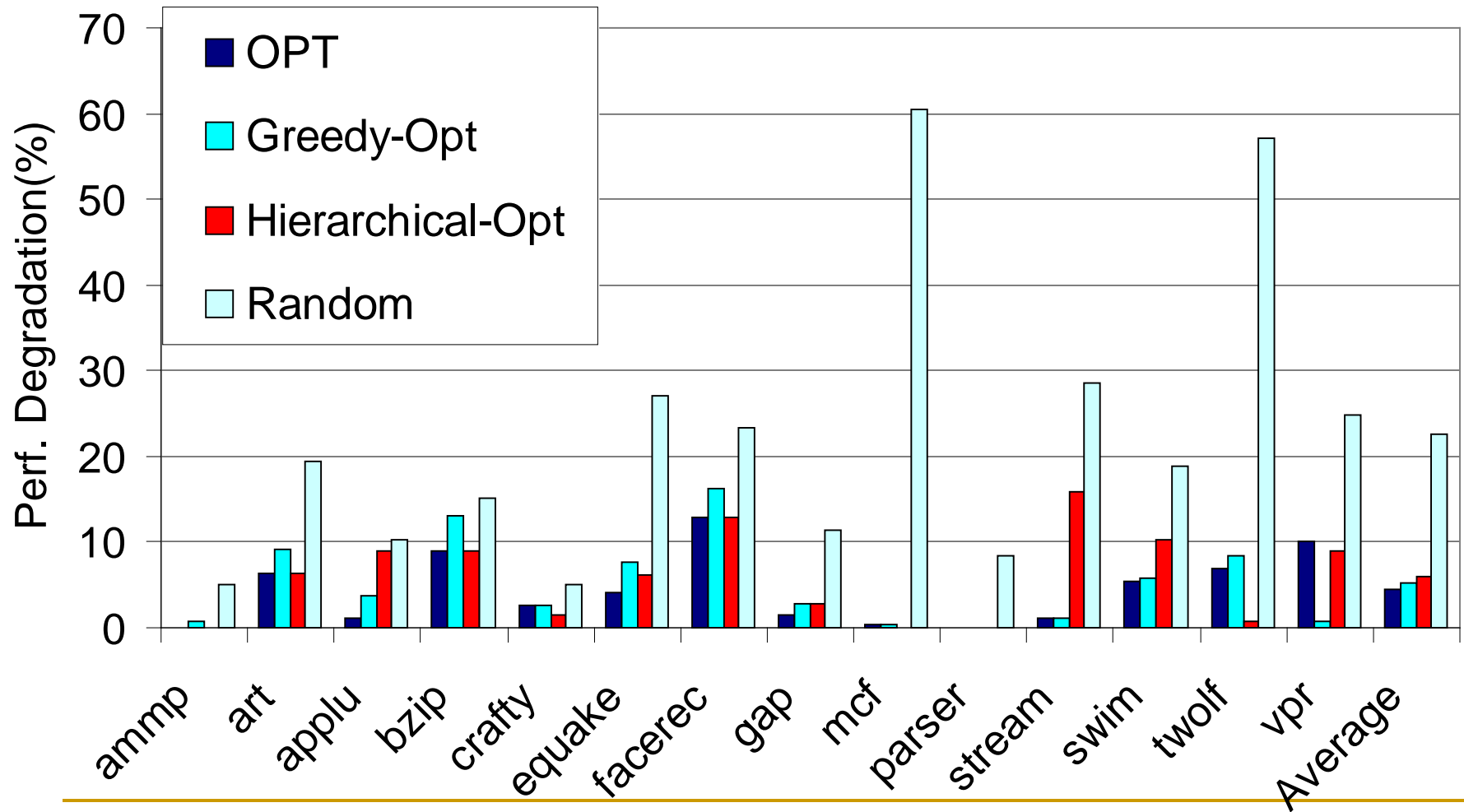
Performance Evaluation

- Machine
 - AMD Opteron 4 core processors
- Benchmarks
 - 15 SPEC CPU2000, 1 Stream
- Metrics
 - Performance Degradation
 - Scheduling time
 - Fairness

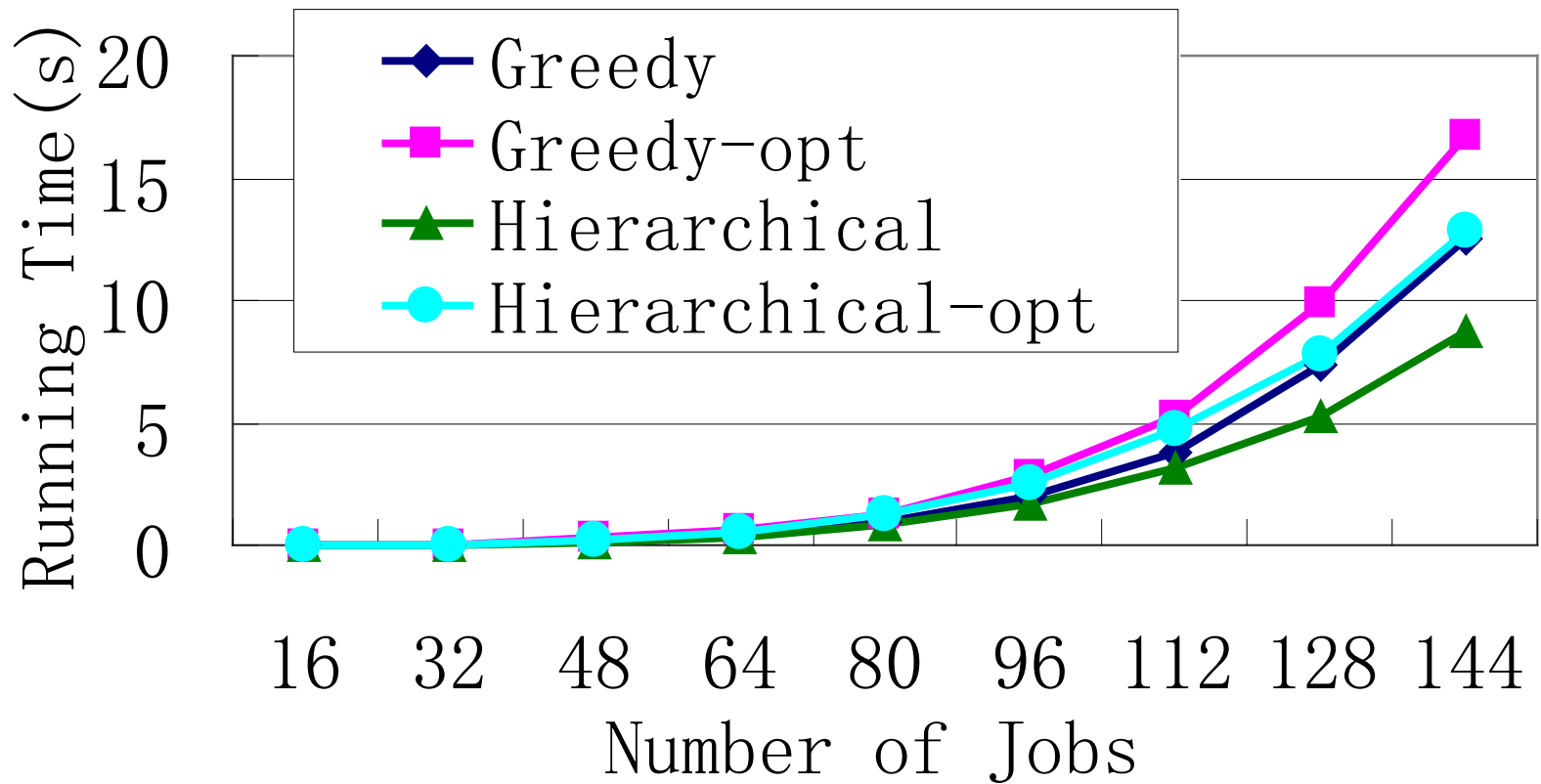
Performance Degradation



Performance Degradation



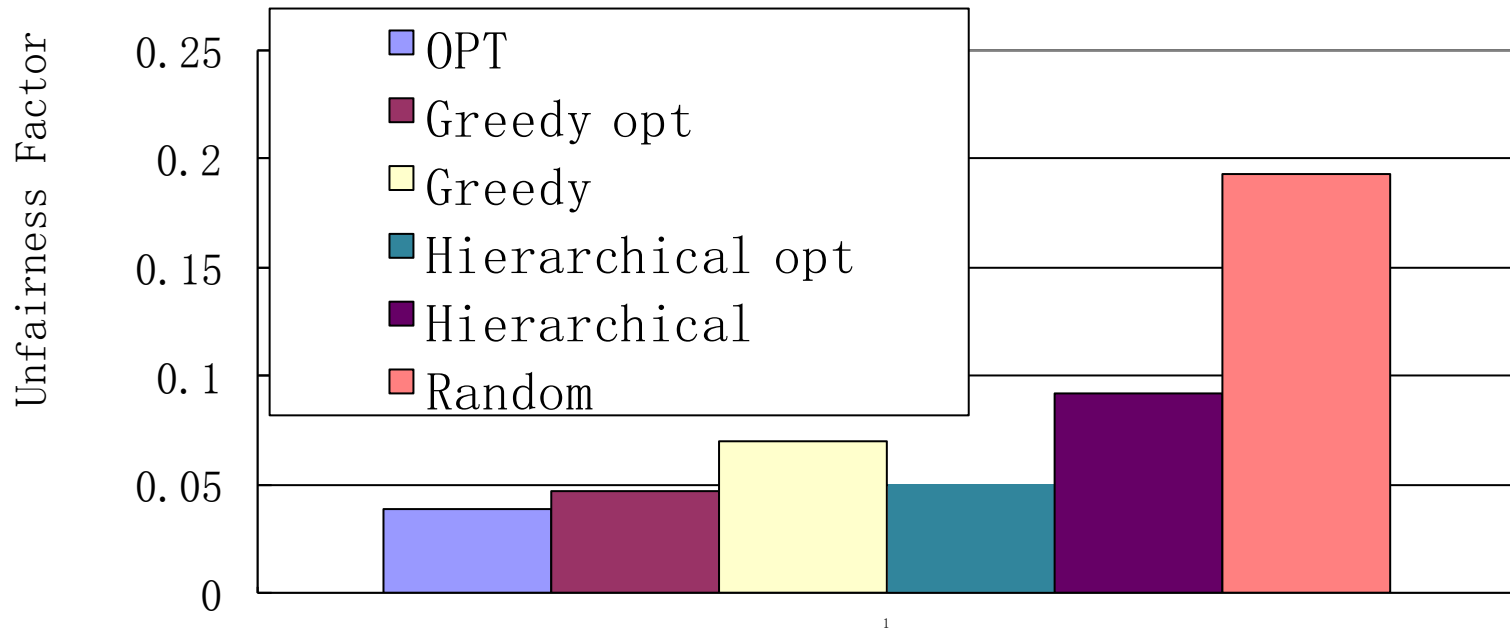
Scheduling Time



Fairness

■ Unfairness Factor

- Coefficient of Variation of normalized degradation



Conclusion

- Job co-scheduling on CMP is crucial
 - Different schedule performance varies
- Dual-core system
 - Polynomial solvable
- K-core ($K > 2$) system
 - NP-Complete problem
 - Heuristics
 - Hierarchical
 - Greedy

Acknowledgement

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Thanks!

Questions?